Engineering Notes

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Probabilistic Combination of Vehicle Dynamic Vibration and Acoustically Induced Random Accelerations

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Introduction

THE development of design load factors for aerospace and aircraft components and experiment support structures, which are subject to simultaneous vehicle dynamic vibration and acoustically generated random vibration, require the selection of a combination methodology. Typically, the vehicle dynamic vibration is low frequency, transient, and deterministic. The random response is typically narrowband random, whose characteristics are described probabilistically with a normal distribution of instantaneous values and a Rayleigh peak distribution. The combination of vehicle dynamic vibration and random response of the single degree-of-freedom system shown in Fig. 1 is illustated in Fig. 2.

Since component support structure design limits generally are defined during the simultaneous occurrence of the two loads, the combination methodology is important from weight and qualification aspects.

The current practice to obtain the combined g level g_c is generally similar to the following.

1) Calculate the vehicle dynamic maximum response.

2) Calculate the G_{rms} response to random input by the Miles relationship and multiply by a factor to get a peak. If a factor of 3 is used, this is the same as the 99.87 percentile level of the instantaneous distribution.

3) Add the vehicle dynamic response and the random peak response on a directional basis to get the combined g_c.

The percentile level for the combined acceleration developed using this practice is not calculable. It would seem, however, that a viable alternate approach would be to determine the characteristics of the probability density function (pdf) of the combined acceleration (vehicle dynamic and random) and select an appropriate percentile level for a combined g_c. The following discussion develops that scenario.

Discussion

If the maximum vehicle dynamic response value is replaced by a sinusoid having an equal maximum value and the same period as the dominant response frequency, then a conservative idealization of the combined response function can be developed, as shown in Fig. 2. The time-history combined

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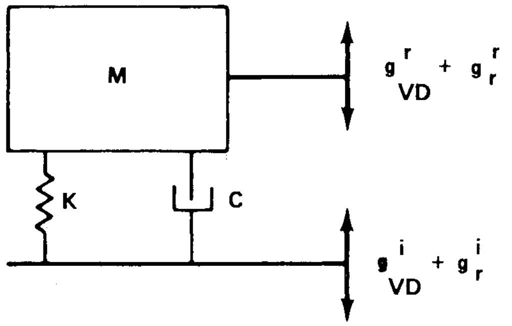

Fig. 1 Single degree-of-freedom component idealization.

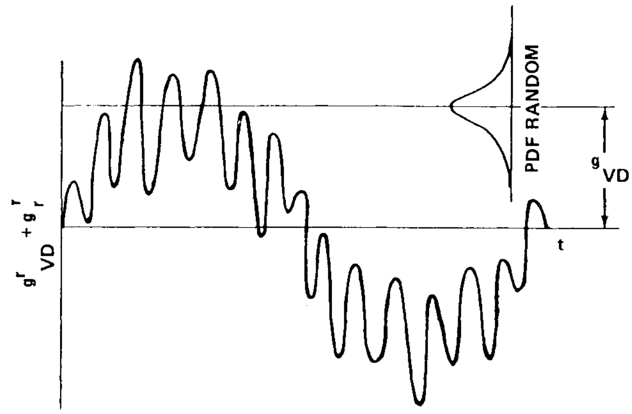

Fig. 2 Time-history combined vehicle dynamic/random response.

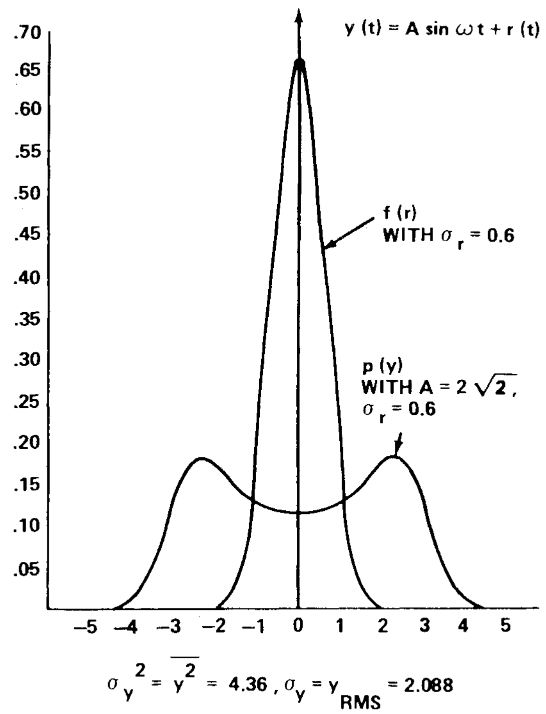

Fig. 3 Combined sinusoidal/random distribution vrs random only, $\sigma_r^2 = .09A2/2$.

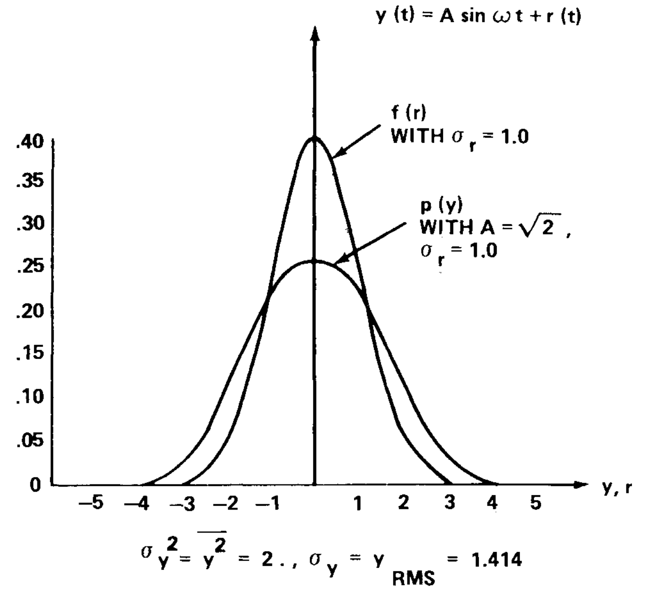

Fig. 4 Combined sinusoidal/random distributions vrs random only, $\sigma_r^2 = .09A2/2$.

functions of Fig. 2 can be idealized as $y(t) = A\sin\omega t + r(t)$, where the randomly varying function $r(t)$ is normally distributed about a zero mean with variance σ_r^2 and a normal pdf of

$$f(r) = \frac{1}{\sigma_r\sqrt{2\pi}} \exp\left(\frac{-r^2}{2\sigma_r^2}\right) \tag{1}$$

Then the pdf of the random function superimposed on the sinusoid (from Refs. 1 and 2) is

$$p(y) = \frac{1}{\pi\sigma_r\sqrt{2\pi}} \int_{-\pi/2}^{\pi/2} \exp\left\{-\frac{1}{2}\left(\frac{y - A\sin\theta}{\sigma_r}\right)^2\right\} d\theta \tag{2}$$

where σ_r is the standard deviation of the random distribution, and A is the peak amplitude of the sinusoid.

Figures 3 and 4 illustrate the characteristics of Eqs. (1) and (2). Figure 3 shows a comparison between the combined pdf and the normal distribution $f(r)$ for a random distribution with a variance σ_r^2 equal to 0.09 of the sinusoidal variance σ_s^2. Obviously, as the random variance approaches zero, the combined distribution approaches a sinusoidal distribution.

Figure 4 shows a comparison between the combined distribution $p(y)$ and the normal distribution $f(r)$ for a superimposed random distribution with a variance σ_r^2 equal to the sinusoidal variance σ_s^2. Obviously, as the random variance gets very large in comparison with the sinusoidal variance, the combined pdf approaches the random pdf.

For a random distribution superimposed on a sine wave, the variance of the combined distribution is

$$\sigma_c^2 = \sigma_s^2 + \sigma_r^2 = A^2/2 + \sigma_r^2 \tag{3}$$

or, the standard deviation of the combined signal equals the rss of the sinusoidal rms and the random rms values

$$\sigma_c = \sqrt{(A^2/2) + (g_{\text{rms}})^2} \tag{4}$$

With the variance of the combined signal and the pdf defined, one can obtain the combined g_c acceleration level appropriate for any selected cumulative probability level.

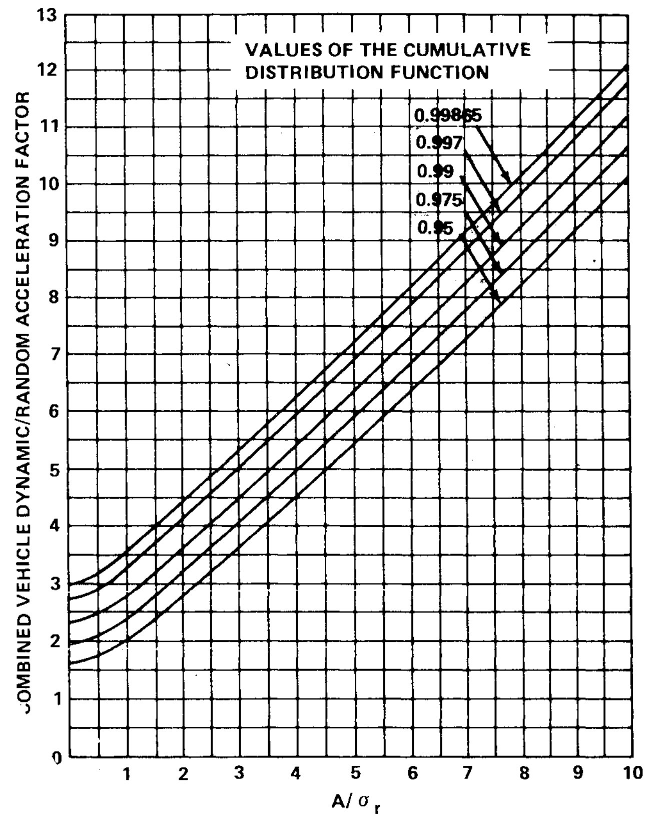

Fig. 5 Combined acceleration g level for selected values of the cumulative distribution function.

The use of this methodology is shown in Fig. 5 for the preselected popular percentile values. The following scenario applies.

1) Determine maximum vehicle dynamics response acceleration A in g's.

2) Determine the g_{rms} response of the article to the random input.

3) Select the combined instantaneous probability level desired (0.95 to 0.99865).

4) Determine the ratio of peak vehicle dynamics response acceleration A to random g_{rms} σ_r.

5) From the appropriate curve, read the ordinate corresponding to A/σ_r and multiply by σ_r to get the quantile pertinent to the combination.

The process will give a combined acceleration (g_c) value for the selected no-exceedance probability.

Summary

In summary, the pdf of a combined sine/random signal has been determined. This, along with the predicted peak vehicle dynamic response accelerations and random response g_{rms}, allows one to describe the combined signal in probabilistic terms. One then can describe g_c to any probability level desired. A rational selection of a combined vehicle dynamic/random response acceleration can now be made for any prescribed probabilistic level.

References

[1]Rice, S. O., *Bell Systems Technical Journal*, Vol. 24, Jan. 1945, pp. 45-156.

[2]Tuell, L. P., Unpublished work, NASA Marshall Space Flight Center, Huntsville, Ala., Dec. 1982.